

ANALYZING SEISMIC SIGNATURES: SDFA FOR DISCRIMINATING EXPLOSIONS AND EARTHQUAKES

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Abstract: *The Smoothed Detrended Fluctuation Analysis (SDFA) method, introduced by Linhares in 2016, offers a valuable tool for characterizing long-range correlations in time series data. Building upon the foundations of Detrended Fluctuation Analysis (DFA) and the wavelet shrinkage procedure, SDFA computes statistical fluctuations measures, denoted as $F(l)$, using varying window lengths (l) . By analyzing these measures across different window lengths, SDFA derives a scaling exponent, specifically the slope coefficient obtained through regression analysis of $F(l)$ against $\ln(l)$, where l varies within a specific range determined by $g(n)$. Linhares, in a subsequent work in 2016, established an optimal choice for $g(n)$, where $g(n)$ is defined as $[(\ln(n))]$ with $[\cdot]$ representing the integer part function and n denoting the length of the time series. This abstract explores the fundamental concepts and procedures underlying the SDFA method, highlighting its significance in uncovering long-range correlations in time series data.*

Keywords: *Smoothed Detrended Fluctuation Analysis (SDFA), time series analysis, long-range correlation, scaling exponent, wavelet shrinkage.*

I. Introduction

Linhares (2016a) propose the Smoothed Detrended Fluctuation Analysis method (SDFA) that describes the long-range correlation of time series. The Detrended Fluctuation Analysis (see Peng et al., 1994) and the wavelet shrinkage procedure (see Donoho and Johnstone, 1994, Donoho and Johnstone, 1995, Donoho et al., 1995 and Vidakovic, 1999) form the basis for the SDFA method. The procedure computes various statistic fluctuations measures $F(l)$, where l represents a window length. Get the wavelet shrinkage estimator $F(l)$ of $F(l)$, for all $l \in \{4, 5, \dots, g(n)\}$. By varying the length l , the $F(l)$ can be characterized by a scaling exponent, more precisely, the slope coefficient of the line obtained by the regression of $F(l)$ on $\ln(l)$, with $l \in \{4, 5, \dots, g(n)\}$. Linhares (2016b) determine an optimal choice for the

number of regressors in the SDFA method given by $g(n) = [(\ln(n))]$, where $[\cdot]$ indicates the integer part function and n is the length of the time series.

Seismic waves are waves of energy arising from human activities, such as a large human-made explosion that gives out low-frequency acoustic energy, or from natural phenomena such as large landslides, magma movement or volcanic eruptions. The presence of the human-made explosion in a seismic catalog may result in errors of statistical analysis of seismicity. One of the major seismological recent challenges is to monitor humanmade explosion arising from mining, road excavating, and other constructional applications. Therefore, the development of methodologies that ensure a correct identification of the type of source generating a recorded seismic signal is, from many perspectives, a very significant and critical issue (see Beccar-Varela et al., 2016).

Different modeling techniques have been developed to discriminate between explosions and natural earthquakes. For instance, Kwang-Hyun (2014) proposed a discrimination method based on the solutions of a double integral transformation in the wave number and frequency domains. Beccar-Varela et al., (2016) investigate the use of wavelets technique as a potential tool to discriminate between natural tectonic earthquakes and humanmade explosions; they investigate and compare the characteristics of the seismic waves generated by a cluster of earthquakes and a set of mining explosions. In Vargas et al. (2017) different waveform-based discrimination parameters were tested using multivariate statistical analysis to develop a real-time procedure for discriminating explosions from earthquakes at regional distances in the Iberian Peninsula. Kahbasi and Moradi (2016) gain new insight into the cross-correlation technique and conduct this approach to discriminate explosions from seismic datasets.

In this work, we analyze the long dependence property in view of the SDFA method (see Linhares, 2016) to compare 7 seismic trace signals of explosions and 7 seismic trace signals of natural earthquakes obtained from astsa R package. It is proposed that utilizing Hurst estimator by SDFA method, like an additional classification tool to discriminate between explosions and natural earthquakes. The paper is organized as follows. Section II describes the Smoothed Detrended Fluctuation Analysis (SDFA) method like an additional classification tool to discriminate between natural tectonic earthquakes and human-made explosions.

In Section III we present the analysis of 7 seismic trace signals of explosions and 7 seismic trace signals of natural earthquakes obtained from astsa R package. Section IV gives the conclusions.

II. SDFA like a Classification Tool

Here we propose the Smoothed Detrended Fluctuation Analysis (see Linhares, 2016a) like an additional classification tool to discriminate between natural tectonic earthquakes and human-made explosions.

Let $\{X\}$ a seismic trace signal. To apply Smoothed Detrended Fluctuation Analysis method to $\{X\}$, it is necessary the following steps. In a first step, a running sum of the observed variable $\{X\}$, is calculated

$$Y = X - \bar{X},$$

for each $t \in \{1, 2, \dots, n\}$, where \bar{X} is the average value of $\{X\}$. In the second step, we divide the time series $\{Y\}$ into $\lfloor n/l \rfloor$ nonoverlapping blocks, where each block has l observations. For each block, one fits a leastsquare line to the data. We denote by Y_t , for $t = 1, \dots, n$, the adjusted fit for each t on each block of length l . After that, we detrend the time series $\{Y\}$, that is, in each block we calculate

$$Z_t = Y_t - Y_t, \text{ for all } t \in \{1, \dots, n\}. \quad (1)$$

In the third step, for each $l \in \{4, 5, \dots, g(n)\}$, we calculate the root mean square fluctuation of the new sequence,

$$F(l) = \sqrt{\frac{1}{n} \sum Z^2} \quad (2)$$

where $n = l \cdot \lfloor n/l \rfloor$.

Two functions are very important in the wavelet analysis: the mother wavelet $\psi(\cdot)$ and the father wavelet $\phi(\cdot)$. These wavelets generate a family of functions that can reconstruct a signal. Given the wavelets $\psi(\cdot)$ and $\phi(\cdot)$, we construct wavelet sequences through translations and dilatations of mother and father

wavelets, respectively, given by $\psi_j(t) = 2^{-j/2} \psi(2^{-j}t - k)$, $\phi_j(t) = 2^{-j/2} \phi(2^{-j}t - k)$.

In general, the most used orthogonal wavelets are: Haar, Daubelets, Symmlets and Coiflets.

The fourth step consists in transform the observations $F(l)$, $l \in \{4, 5, \dots, g(n)\}$, into the symmlet wavelet “s8” domain by applying a discrete wavelet transform (see definition 2.1), with level $J = \lfloor \log(g(n) - 3) \rfloor$, to obtain a sequence of wavelet coefficients \dots, \dots .

Definiton 2.1 (Discrete Wavelet Transform). Let $(X_1, X_2, \dots, X_n)'$ be an i.i.d. random sample, with $J = \lfloor \log(n) \rfloor$, where $\lfloor \cdot \rfloor$ indicates the integer part function. The discrete wavelet transform (DWT) of $\{X\}$, with respect to the mother wavelet $\psi(\cdot)$, is defined as

$$d_j = \sum_{k=1}^n \psi_k(t) \quad (3)$$

for all $j = 1, 2, \dots, J$ and $k = 1, 2, \dots, n$. We can write the transform (3) in matrix form by

$$D = \Psi \quad (4)$$

where $\Psi = [\psi_k(t)]$ is a $J \times n$ matrix. Assuming appropriate boundary conditions, the transform is orthogonal, and one can obtain the inverse discrete wavelet transform (IDWT) given by

$D' = D^{-1}$ where $'$ denotes the transpose of.

Then shrink the wavelet coefficients towards zero, to obtain new detail coefficients $d \equiv \delta(d_1), \dots, d \equiv \delta(d_n)$, where $\lambda = \sigma \sqrt{2 \log(g(n) - 3)}$, σ is the estimated level of noise given by

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$$\sigma = \frac{\text{median}\{d_k : 0 \leq k < 2\}}{0.6745}$$

and the $\delta(x)$ is the hard (H) shrinkage function defined by

$$\delta(x) = \begin{cases} 0, & \text{if } |x| \leq \lambda \\ x, & \text{if } |x| > \lambda \end{cases}$$

Finally, apply the inverse discrete wavelet transform, to get the wavelet shrinkage estimator $F(l)$ of $F(l)$, for all $l \in \{4, 5, \dots, g(n)\}$.

Under such condition, the smoothed fluctuations can be characterized by a scaling exponent H , which is the slope of the line when one regresses $\ln(F(l))$ on $\ln(l)$,

$$F(l) \sim \phi l^H \quad (5)$$

By taking the logarithm of the relationship in (5), we obtain $\ln F(l) \sim \ln(\phi) + H \ln(l)$. Then by the least squares method, the estimator for the exponent H is given by

$$H = \frac{\sum_{l=1}^n (\ln(l) - \bar{x})(\ln(F(l)) - \bar{y})}{\sum_{l=1}^n (\ln(l) - \bar{x})^2} \quad (6)$$

where.

$$\bar{x} = \frac{1}{n} \sum_{l=1}^n \ln(l)$$

Linhares (2016b) determine an optimal choice for the number of regressors in the SDFA method given by $g(n) = \lfloor (\ln(n)) \rfloor$, where $\lfloor \cdot \rfloor$ indicates the integer part function and n is the length of the time series.

We proposed the Hurst estimator H by the Smoothed Detrended Fluctuation Analysis method (SDFA) like an additional classification tool to discriminate between natural tectonic earthquakes and human-made explosions, where if

- $H > 0.4$ then $\{X\}$ is a seismic trace signal of natural earthquake, and if
- $0 < H \leq 0.4$ then $\{X\}$ is a seismic trace signal of explosion.

III. Application

Seismic catalogs often include human-made contamination, for instance, quarry explosions and marine shots. Seismologists assessing seismicity of a region are frequently confronted with the challenge of identifying and excluding artificial events from seismic catalogs. A contaminated seismic catalog is a major potential source of errors and falsifies seismicity rates in a region under investigation (see Kahbasi and Moradi, 2016). In this Section, we analyze the long dependence property in view of the SDFA method (see Linhares, 2016a) to compare 7 seismic trace signals of explosions and 7 seismic trace signals of natural earthquakes obtained from *astsa* R package in order to check the difference between explosions and natural earthquakes.

The Hurst parameter $H()$ is used to quantify long range dependence in time series data, here we consider the SDFA method (ver Section II) to estimate the parameter H . It is proposed that utilizing Hurst

estimator by SDFA method, like an additional classification tool may provide an indication of difference between explosions and natural earthquakes. Table 1 presents the Hurst estimates H by SDFA method with $g(n) = \ln(n)$, for the all seismic trace signals from astsa R package.

From Table 1 one observes that for seismic trace signals of natural earthquakes the estimates of H is always bigger to 0.4. While for seismic trace signals of explosions the estimates of H is always less or equal than 0.4. For each seismic trace signals, this conclusion is statistically significant at 1% significance level. Therefore, there is a numerical evidence that for seismic trace signals of natural earthquakes we have $H \in (0.4, 1)$, while for seismic trace signals of explosions the $H \in (0, 0.4]$.

The figure 1 shows the graphical representation for the Table 1 values. The figure 2 shows the fluctuation functions (in log-scale) of the SDFA method for the seismic trace signals of natural earthquakes of number EQ4, where the slope of the regression line is bigger than $H = 0.4$. Figure 3 shows the fluctuation functions (in logscale) of the SDFA method for the seismic trace signals of explosion of number EX4 and we can note that the regression line is smaller than $H = 0.4$.

Table1. Estimation results by SDFA method for the parameter in all seismic trace signals from astsa R package

Natural Earthquakes		Explosions	
Record's Number	H	Record's Number	H
EQ1	0.4216	EX1	0.3055
EQ2	0.4886	EX2	0.2672
EQ3	0.5923	EX3	0.4000
EQ4	0.6345	EX4	0.1936
EQ5	0.9720	EX5	0.1728
EQ6	0.7460	EX6	0.1616
EQ7	0.4493	EX7	0.3951

IV. Conclusions

It is proposed that utilizing Hurst estimator by SDFA method, like an additional classification tool to discriminate between explosions and natural earthquakes, where if $H > 0.4$ then $\{X\}$ is a seismic trace signal of natural earthquake and if $0 < H \leq 0.4$ then $\{X\}$ is a seismic trace signal of explosion. We analyze the long dependence property in view of the SDFA method to compare 7 seismic trace signals of explosions and 7 seismic trace signals of natural earthquakes obtained from astsa R package in order to check the difference between explosions and natural earthquakes.

We saw a numerical evidence that for seismic trace signals of natural earthquakes we have $H \in (0.4, 1)$, while for seismic trace signals of explosions the $H \in (0, 0.4]$.

Figure2. Plot Presenting the Scaling Properties of SDFA in -Scale for the Seismic Trace Signal of Natural Earthquakes EQ4.

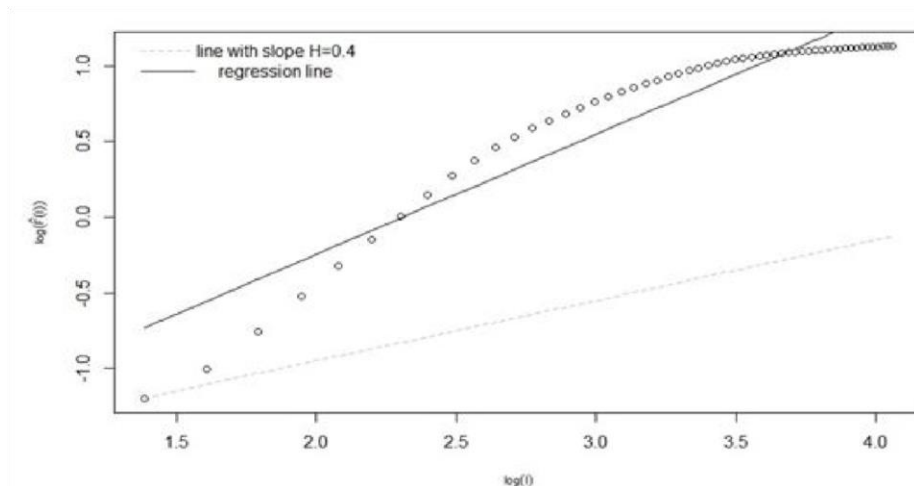
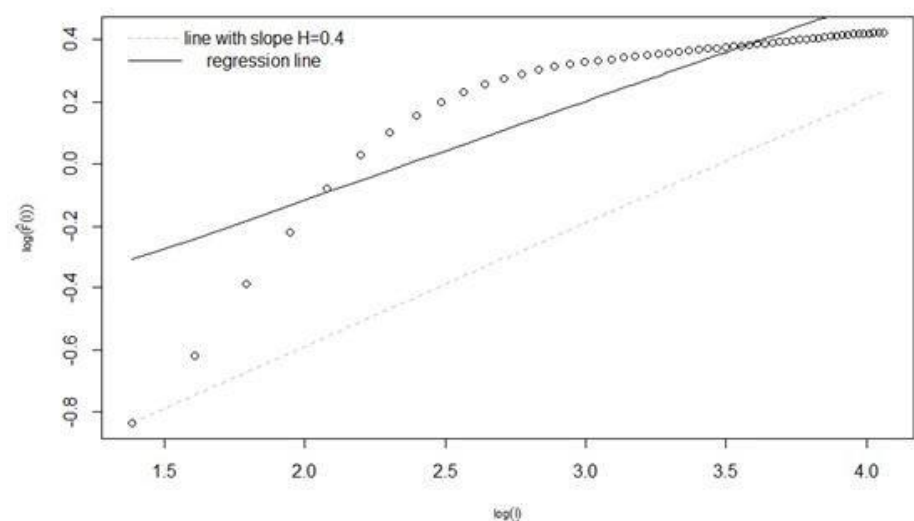


Figure 3.Plot Presenting the Scaling Properties of SDA in Signal of Explosion EX4.

-Scale for the Seismic Trace



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